12

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THEODOLITE ACCURACY STATISTICAL ANALYSIS METHODOLOGY

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This report discusses the THEODOLITE computer program and algorithm. THEODOLITE is a FORTRAN program designed to provide information, based upon probability theory, on the reliability of the intersection data and its corresponding estimate of bullet splashdown locations calculated by the DERANGE program. The DERANGE program takes angular theodolite readings and computes estimated bullet impact points. DERANGE is useful for determining ammunition quality. The THEODOLITE algorithm is useful as a reliability check of DERANGE or any program giving the same type of data as DERANGE. The THEODOLITE program works not only for the sample median estimate of impact provided by DERANGE but also for any estimate based upon the locations of the intersections generated by theodolites. Several rounds must be fired in order to use this program since it relies on an approximate sample variance of theodolite angles derived from all rounds fired.

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FOREWORD

Ammunition is often tested at the Naval Surface Warfare Center, Dahlgren Division (NSWCDD) gun range by firing a number of rounds into the Potomac River Range and observing the locations of the splashdown points. The impact locations are estimated by three or four theodolites, located at predetermined places on the river banks. These theodolites are used by observers to measure the rounds' angles of impact. Intersections of the rays of these angles represent measurements of the projectile impact locations. These measurements are used to compute an estimate of the true impact points. The program at NSWCDD that takes the angular theodolite readings and computes the estimated impact points is called DERANGE.

The theodolite analysis program called THEODOLITE was created to help determine the accuracy of the point-of-impact estimates as derived by DERANGE. THEODOLITE gives the approximate probability that typical range and drift measurements using the theodolites are within a given distance from the impact estimates provided by DERANGE. The idea is that the more reliable the estimate, the higher the probability that typical theodolite measurements fall within a certain distance from the estimate. This is because reliability decreases the variance of theodolite measurements, therefore causing the increase in probability.

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CONTENTS

INTRODUCTION

The Naval Surface Warfare Center, Dahlgren Division (NSWCDD) often tests ammunition by firing a number or rounds into the Potomac River and observing the locations of the splashdown points. The impact locations are estimated by three or four theodolites, located at predetermined places on the river banks. These theodolites are used by observers to measure the angles of impact. Intersections of the angles' rays represent measurements of the projectile impact locations. These measurements are used to compute an estimate of the true impact points. The DERANGE program takes the angular theodolites readings and then computes the estimated impact points. It is useful for determining ammunition quality.

The theodolite analysis program called THEODOLITE was created to help determine the accuracy of the point of impact estimates as derived by DERANGE. THEODOLITE gives the approximate probability that typical range and drift measurements using the theodolites are within a given distance from the impact estimates that DERANGE provided. The concept is that the more reliable the estimate, the higher the probability that typical theodolite measurements fall within a certain distance from the estimate. This is because reliability decreases the variance of theodolite measurements, therefore causing the increase in probability.

OVERVIEW OF THE DERANGE SPLASHDOWN LOCATION PROGRAM

The DERANGE program requires perceived angles of impact from n different theodolites, the theodolite locations, and the gun location. A coordinate transformation is made from the north and east frame, (where x is east and y is north) to a range and drift frame. A translation of the coordinate system is also made so that the gun is situated at the origin of this system.

The program algorithm calculates the intersection of the lines formed by the theodolite angles. Figure 1 shows three theodolites on the same side of the river. The intersections formed by the three theodolite perceived angles of impact are (x,y), (p,q), and (r,s).

After calculating the intersections, the program algorithm then realigns both the abscissa and the ordinate axes in ascending order. If the number of intersections is odd, the middle entry of the X-axis and the middle entry of the Y-axis are chosen to represent the estimated splashdown point. Hence, for the X-axis, the number of entries less than the middle entry equals the number of entries greater than the middle entry. (The same is true for the middle Y-axis entry.) If the number of intersections is even, the average of the middle two entries on the X-axis represents the estimated splashdown. (The same is true for the middle Y-axis entry.)

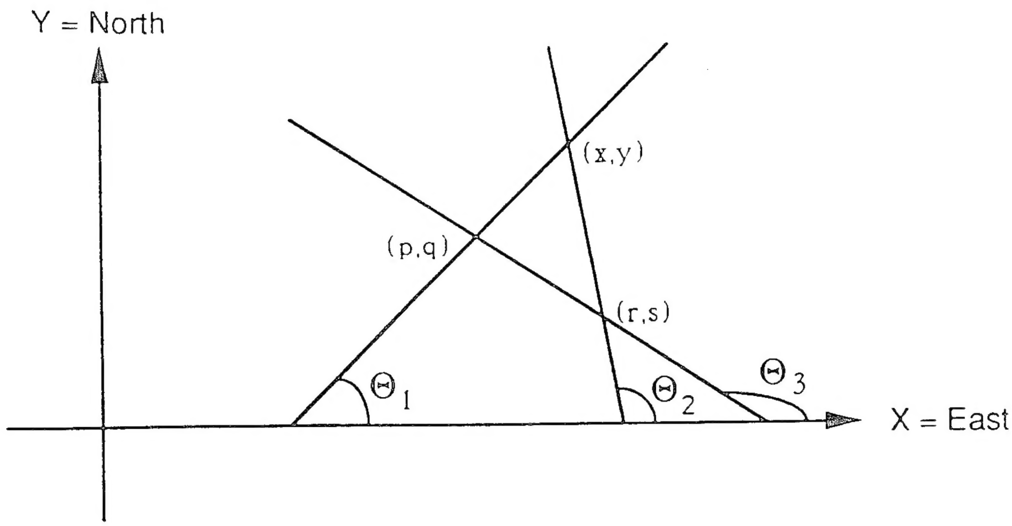

FIGURE 1. READINGS FROM THREE THEODOLITES

For the purpose of this report, the x and y entries generated by this method to represent the estimated splashdown point are called the sample medians. Note that the sample median in this algorithm differs somewhat from the sample median defined in conventional statistical theory because each sample originates from a different probability distribution.

The program calculates the deviation in the sample median. The deviation is the average distance from the sample median to each of the intersections. Hence, this is the average deviation in the X-axis and Y-axis between the estimated bullet splashdown point and the intersection of the lines formed by the theodolite angles.

The output of DERANGE is in the range/drift frame, where the X-axis is the range (instead of East) and the Y-axis represents drift.

The program is useful for judging the quality of ammunition lots since it estimates impact locations of the rounds fired. Impact locations indicate the dispersion of the rounds, which is useful in determining the quality of the ammunition lot.

OVERVIEW OF THE THEODOLITE ANALYSIS PROGRAM

The THEODOLITE program estimates the probability that the intersections computed from observed theodolite angles are within a certain distance in range and drift from the sample median that DERANGE calculated for a given round of gunfire. This program can also be used to

approximate the probability that intersections, computed by theodolite angles, are within a given square centered at the sample median of a given round of bullet impact. The THEODOLITE algorithm is based on the assumptions that the observed angles of the theodolite are normally distributed, where the means of the angles are computed from the sample median, and the variances of the angles are calculated from the intersections over all rounds fired. Several rounds must be fired in order to compute this variance estimate.

This program is useful in helping to determine the reliability of using the bullet impact estimate, namely the sample median, calculated by DERANGE. The estimate's reliability increases proportionately with the increasing probability that theodolite angles are within a given distance from the estimated range and drift of the impact.

EFFECTS OF ACCURACY

Inaccuracy of the sample median as an estimate of the impact of a given round has three main causes:

- The possible presence of outliers, where the data contain a large accidental error in the theodolite angular measurements.
- The angles formed by the lines of intersection of theodolite angle rays are excessively small.
- The theodolite measurements are inaccurate.

These three situations cause the variance of the observed intersections of the theodolite measurements to be relatively large. A large variance indicates that the observed intersections, (i.e., the observed impacts) may vary widely. As a result, the estimate of the impact as given by the sample median of the intersections may be unreliable. This large variance cause the probabilities derived by THEODOLITE to be small.

PROGRAM REQUIREMENTS

The THEODOLITE Analysis program requires the following data:

- The x and y intersections in the range and drift coordinates of the theodolite line of sight from DERANGE.
- The sample medians as input from DERANGE.
- The gun and theodolite locations in the northeast (NE) frame.
- The round number of the gun shots where analysis is desired.
- The length of the probability confidence square that contains the sample median.
- The gun angle of fire from the north.

STATISTICAL ANALYSIS METHODOLOGY

Given n rounds of gun shots, a coordinate transformation is performed on each intersection from the range/drift frame to the NE frame. A translation is also performed from the gun location to the origin of the NE frame. For each intersection, two theodolite angles are derived. The corresponding theodolite pointing angles for the sample median are computed; these angles are considered to be the means of the theodolite angles. Note that the mean for each theodolite is different for each of the rounds. The equations used to compute theodolite angles for either the intersection or sample median are as follows:

$$\Theta_i = \text{Arctan}\left(\frac{y-y_i}{x-x_i}\right),\ \hat{\Theta}_i = \text{Arctan}\left(\frac{\hat{y}-y_i}{\hat{x}-x_i}\right)\quad i = 1,2,3,4 \tag{1}$$

where

Θ_i = ith theodolite angle

$\hat{\Theta}_i$ = ith theodolite angle for the sample median

(x, y) = intersection of rays from two theodolites

(x_i, y_i) = location of ith theodolite

$(\hat{x}, \hat{y})$ = sample median for a given round

From the computed theodolite angles, the approximate sample variance of each theodolite angle over the n rounds is calculated by Equation (2), where:

$$\hat{\text{Var}}\,(\Theta_i) = \sum_{k=1}^{n} [(\Theta_i(k) - \hat{\mu}_i(k)) - \overline{(\Theta_i(k) - \hat{\mu}_i(k))}]^2/(n-1)$$

$$= [n \sum_{k=1}^{n} (\Theta_i(k) - \hat{\mu}_i(k))^2 - (\sum_{k=1}^{n} (\Theta_i(k) - \hat{\mu}_i(k)))^2]/n\,(n-1) \tag{2}$$

i = 1,2,3,4

$\hat{\mu}_i(k)$ = estimated mean of Θ_i at round k = $\hat{\Theta}_i$

For Equation (2), the computed sample variance is identical for every round for each theodolite because it is calculated by using the results of all rounds fired.

The theodolite angle readings from each theodolite are modeled as being normally distributed. Each mean is calculated from the sample median. Each variance is calculated from the sample variance. Note that each theodolite has one computed sample variance and one computed mean. The program assumes that either three or four theodolites are used and that each theodolite is statistically independent of every other theodolite.

Treating theodolite readings as independent normally distributed random variables, the distribution of one of the intersections from any two theodolites can be derived by using the theory of transformations of random variables. However, given the distribution of the X and Y axis (East and North) random variables of one of the intersections, the derivation of the distributions of additional intersections is a very difficult problem.

Difficulty occurs because, although the random variables representing theodolite angles are independent, the (X,Y) random vector of each intersection is statistically dependent on the (X,Y) random vector of every other intersection. This dependence occurs because each theodolite angle random variable is used to determine several intersections and, therefore, the intersections must be related to each other. The statistical dependency greatly complicates the mathematical derivation of the distributions of additional intersections; their distributions are presently unknown. Hence, instead of using the exact distributions, the probabilities associated with each intersection are computed using the Monte Carlo simulation method.

To compute the probability that the range and drift axis of intersections falls within a given distance from the range and drift axis of the sample median by the simulation method, the observation angles of the theodolites are first simulated as being normally distributed. The sample variance is computed for each theodolite from the observed intersections over the n rounds. This variance is considered to be the variance of the normal distribution. The assumed mean for each theodolite as calculated from the observed intersections for a given round is considered to be the mean of the theodolite for the given round.

After simulating normally distributed angles for each theodolite, corresponding intersections from pairs of these angles are calculated in the NE frame by the following formula for four theodolites, where x represents east and y represents north:

$$X = \frac{y - b_j}{\tan \Theta_j}, \quad Y = \frac{b_i \tan \Theta_j - b_j \tan \Theta_i}{\tan \Theta_j - \tan \Theta_i}$$

$$b_j = y_i - x_i \tan \Theta_i, \quad b_j = y_j - x_j \tan \Theta_j \tag{3}$$

(b_i, b_j) = pair of y intercepts for the lines from the pair of angles
(x_i, y_i) = location of theodolite i
(x_j, y_j) = location of theodolite j
i, j = 1,2,3,4; $i \neq j$

If three theodolites are used, three intersections can be formed. For four theodolites, six intersections are formed. Figure 2 shows readings for four theodolites located on the same side of the river.

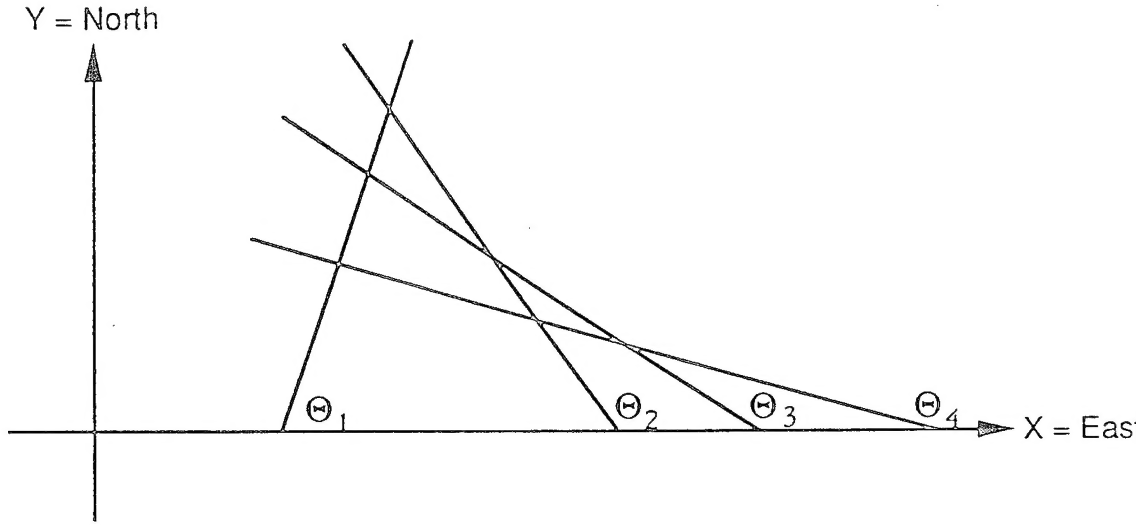

FIGURE 2. READINGS FROM FOUR THEODOLITES

A coordinate transformation is performed in the intersections from the NE frame to the range and drift frame. Included in this transformation is a translation so that the origin is the gun location.

The range and drift coordinates of each intersection derived from the simulated angles are tested to see whether they are within an input distance from the range/drift coordinates of the sample median. There is a counter for the range and drift of each intersection. If the range/drift of the simulated intersections falls within this distance, a 1 is added to a counter. For three theodolites, there are three intersections and therefore six counters (three for range and three for drift). For four theodolites, there are six intersections and therefore twelve counters. This simulation of intersections via simulated theodolite angles based on normal distributions is repeated a specified number of times. Replicating 10000 times is sufficient for most purposes. The probability that the range/drift falls within the specified input distance from the sample median is computed by dividing each counter by the total number of replications.

Hence, the program determines the probabilities that typical intersections from each pair of theodolite measurements fall within a specified confidence distance from the sample median. These probabilities are used to help judge the reliability of estimates from DERANGE or similar programs.

THEODOLITE ANALYSIS PROGRAM INPUT AND OUTPUT

The THEODOLITE program is written in FORTRAN and hosted on a VAX computer. It requires two input files plus supplemental data. These files contain the necessary information to run the program. Another file is required to provide output. This section describes the three files and the required supplemental data.

FILE 1

One of the files contains the theodolite intersections provided by the DERANGE program. This file is assigned to file 1 in the program. The intersections (see Table A-1) are entered in file 1. In the case where three theodolites are used, there exists three possible intersections. The theodolites are aligned in ascending order of numbers assigned to the theodolites in the test range. For simplicity in description, the ordered theodolites are called 1, 2, and 3, respectively. The three intersections are noted by ordered pairs with numbers corresponding to the theodolites that gave the intersections. For example, the intersections from theodolite 1 and 2 are denoted by (1,2). For three intersections, file 1 consists of the values of these intersections, in the following order: (1,2), (1,3), (2,3).

For the case of four theodolites, the test range numbers are again arranged in ascending order and renamed 1, 2, 3, and 4. The ordered pairs of the intersections are named in the identical manner as the three theodolites. The values of these intersections are given in the file in the following order: (1,2), (1,3), (2,3), (2,4), (3,4), and (1,4). Recall that six intersections are generated by four theodolites. The values for the range axis are given first in file 1, and one row of data is entered for each round of gun fire. After the range axis values are given, the drift axis values are entered in the same way.

The program is written to read six entries for the range and six entries for the drift. Hence, in the case of three theodolites, generating only three entries for each axis, zeroes must be entered for entries 4 through 6. For example, in Table A-1, although four theodolites were used, data from the fourth theodolite were missing in round 5, and therefore zeroes were placed in entries 4 through 6 of round 5.

FILE 2

The other file supplies miscellaneous information, including the sample medians from DERANGE, that are also essential to the algorithm. This file is assigned to file 2. The first line contains the gun location in the NE frame. (See Table A-2) Since the East axis is considered to be the X axis and North is the Y axis, the East values in yards must be entered first, followed by the value in yards in the North. The next n lines consist of the range and drift entries, respectively, of the estimated point of impact, which are the sample medians. The n is the number of rounds being considered. The next line contains the NE frame's East axis location of each of the theodolites used.

If only three theodolites were used, a zero must be placed in the fourth entry. The last line contains the locations of each theodolite in the North axis of the NE frame. As with the East axis, a zero must be placed in the fourth entry if only three theodolites were used.

SUPPLEMENTAL DATA

Along with file 1 and file 2, some information must also be supplied interactively. The program writes the questions to be answered on the terminal and pauses for the reply. The questions are self explanatory. The needed information is as follows:

- Number of rounds.
- Number of cuts (intersections) 3 or 6.
- Angle of gun line of fire in degrees from the North.
- The round number for which analysis via this program is desired.
- The confidence interval distance from the sample median that is desired
- The number of replications desired (See Figure A-4).

FILE 8

The output of the program first gives the gun line of fire from North as input. Next, the gun location (yards) in the range/drift frame is given as input followed by the estimated impacts in both the range/drift frame and the NE frame, where x represents East and y represents North. This is followed by the theodolite location in x and y. The intersections in x and y are then given. This is followed by the sample variances of the theodolites over all rounds and the computed probabilities that the intersections fall within the input confidence distances from the sample median. For both the range and drift, the probabilities are ordered the same way as the input intersections. The output is written on file 8. The sample variances and computed probabilities are also written directly to the user's terminal. Figure A-3 shows a sample output in report format.

MISSING DATA

For theodolite pairs with missing intersection data, zeroes are written in the part of the output showing the intersection in the range/drift frame. For example, in Figure A-3, intersection data were missing in round 5 for the theodolites pairs, (2,4), (3,4), and (1,4). Hence, zeroes were written for these combinations in the range/drift frame. Since zeroes are written in the range/drift frame, the impacts would be considered to be at the gun location itself. Hence, the values 62.720 and 34.745, being the gun location in the NE frame were written in the output for this intersection with respect to the NE frame. This also represents the coordinate transformation and translation of (0.,0.) from the range/drift frame to the NE frame.

The THEODOLITE program will not run unless files 1, 2, and 8 are properly assigned to match the correct files (as described above). The yard is the unit of length used in all inputs and outputs of THEODOLITE. An example of the command procedure is provided below:

- $ASSIGN THEODOLITE.DAT FOR001
- $ASSIGN THEODOLITE.IN FOR002
- $ASSIGN THEODOLITE.OUT FOR008
- $RUN THEODOLITE

SAMPLE INPUT/OUTPUT

Appendix A tables show a sample input of the THEODOLITE program. The current FORTRAN code for the THEODOLITE program is provided in Appendix B.

Figure 3 shows a diagram of the locations of the theodolites, gun, and sample median of the sample run. The gun is at the origin of the (X,Y) coordinates. The X-axis is East and the Y-axis is North.

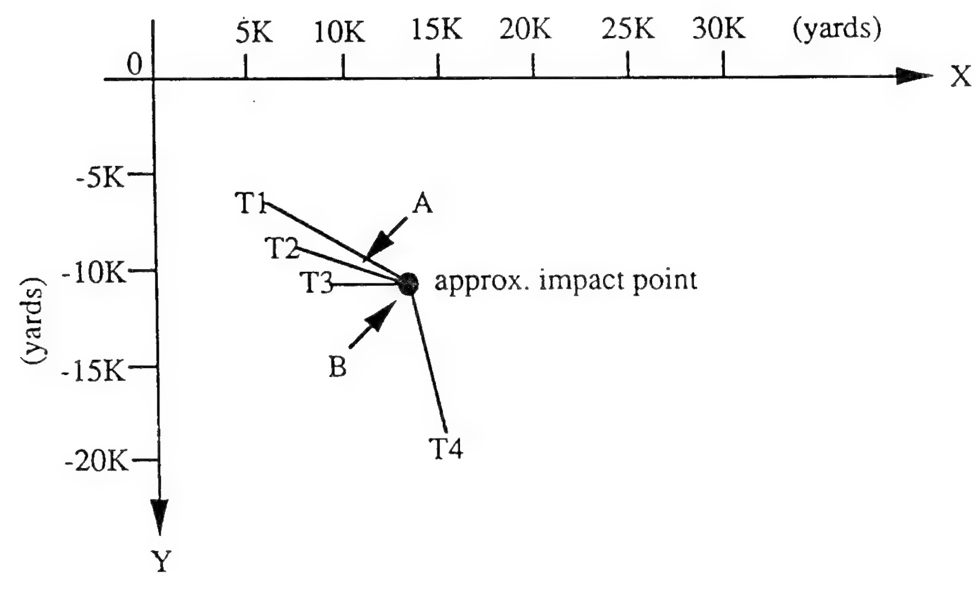

FIGURE 3. LOCATION OF FOUR THEODOLITES
AND APPROPRIATE IMPACT POINT

Note that angle A, formed near the sample median by the intersections of rays that are generated by observed theodolite angles 1 and 2, is much smaller than angle B. Angle B is generated by theodolites 3 and 4 in exactly the same way as angle A. Recall that these intersections were called (1,2) and (3,4), respectively.

The amount of variation of an intersection because errors in theodolite measurements is inversely related to the size of the intersection angle. Hence, any change in theodolite readings from theodolite 1 or 2 causes a greater variation in the (1,2) intersection than in that of (3,4). This would cause the probability of finding the intersection within a specified distance from the sample median in any coordinate system for (1,2) to be smaller than that of (3,4). The range/drift probabilities for (1,2) are, respectively, 0.1406 and 0.0482; the same for intersection (3,4) are 0.9917 and 0.9532. This is consistent with the theory as explained above. Hence, one can conclude that the (1,2) data, having a much lower probability, is not as reliable as the (3,4) data.

A threshold value of defining low and high probability has not yet been established because of the complexity as well as subjectivity of the problem. Perhaps a threshold can be determined empirically by running THEODOLITE on past outputs of DERANGE, where estimates in some outputs have proven to be accurate and others to be inaccurate. For example, if estimates known to be poor gave probabilities in range for a given confidence distance near 0.40, while estimates known to be good gave probabilities of approximately 0.80, perhaps a threshold of 0.60 could be used for the particular confidence distance.

If the probabilities of all six intersections seem low, one can conclude that all of the intersection data may be unreliable. Unreliable data can be caused by the angle problems as explained in the previous paragraph, statistical outliers, or inaccuracies of the theodolites measurements. If these problems exists, the experiment may need to be repeated where the angle problems, outliers, or theodolite accuracy problems are eliminated in order to obtain new data.

In a situation where four theodolites, generating six intersections, are used in some but not all of the rounds fired (e. g., round 5 of the sample run), the program still calculates the estimated variances for all four theodolites using all available data. For example, the variance for theodolite 4 in the sample run is calculated using only nine rounds. In this situation probabilities for all six intersections are still calculated for all rounds. This is mathematically correct since the probabilities are derived from simulated normally distributed theodolite angles, and any normal distribution can be characterized by the mean and variance, which are available for all rounds. The confidence probabilities for the rounds with missing theodolite angles and therefore with missing intersections can be interpreted as the "confidence had the data been present." If only three theodolites are used and the same ones are used for all the rounds, the user must so indicate on the terminal input and; the confidence probabilities of only three intersections will be given in the output.

SUMMARY

The THEODOLITE program was designed to provide information, based upon probability theory, on the reliability of the intersection data and its corresponding estimate of bullet splashdown locations calculated by the DERANGE program. This algorithm is useful as a reliability check of DERANGE or any program giving the same type of data as DERANGE. This program works not only for the sample median estimate of impact provided by DERANGE but also for any estimate based upon the locations of the intersections generated by theodolites. Several rounds must be fired in order to use this program since it relies on an approximate sample variance of theodolite angles derived from all rounds fired.

APPENDIX A

THEODOLITE PROGRAM SAMPLE INPUT

TABLE A-1. INTERSECTION DATA INPUT BY THEODOLITE STATION COMBINATION

Range (yd)						
Round	(1,2)	(1,3)	(2,3)	(2,4)	(3,4)	(1,4)
1	16866.54	16808.21	16788.18	16823.82	16816.21	16819.43
2	16714.24	16645.80	16622.41	16619.65	16620.27	16609.43
3	16906.96	16858.87	16842.28	16848.82	16847.44	16842.90
4	16833.48	16768.04	16745.57	16773.26	16767.30	16767.00
5	16945.47	16886.78	16866.56	0.00	0.00	0.00
6	16930.93	16864.97	16842.28	16870.59	16864.62	16864.48
7	16866.54	16830.80	16818.45	16837.46	16833.43	16834.48
8	16964.90	16874.90	16844.09	16885.15	16876.50	16877.13
9	17001.86	16939.54	16918.08	16960.92	16952.04	16956.87
10	16817.23	16829.68	16834.01	16844.88	16842.58	16847.73

Drift (yd)						
Round	(1,2)	(1,3)	(2,3)	(2,4)	(3,4)	(1,4)
1	22.60	41.45	56.54	41.10	35.43	37.83
2	10.57	33.22	51.42	52.65	53.09	45.25
3	12.14	27.67	40.09	37.26	36.23	32.83
4	-0.36	21.06	38.20	26.05	21.63	21.40
5	4.94	23.86	38.93	0.00	0.00	0.00
6	1.78	23.10	40.09	27.86	23.37	23.26
7	22.60	34.15	43.42	35.19	32.18	32.96
8	1.31	30.29	53.26	35.60	29.09	29.57
9	21.02	40.85	56.61	38.41	31.66	35.33
10	46.27	42.26	39.01	34.31	32.59	36.44

16866.54	16808.21	16788.18	16823.82	16816.21	16819.43
16714.24	16645.80	16622.41	16619.65	16620.27	16609.43
16906.96	16858.87	16842.28	16848.82	16847.44	16842.90
16833.48	16768.04	16745.57	16773.26	16767.30	16767.00
16945.47	16886.78	16866.56	0.00	0.00	0.00
16930.93	16864.97	16842.28	16870.59	16864.62	16864.48
16866.54	16830.80	16818.45	16837.46	16833.43	16834.48
16964.90	16874.90	16844.09	16885.15	16876.50	16877.13
17001.86	16939.54	16918.08	16960.92	16952.04	16956.87
16817.23	16829.68	16834.01	16844.88	16842.58	16847.73
22.60	41.45	56.54	41.10	35.43	37.83
10.57	33.22	51.42	52.65	53.09	45.25
12.14	27.67	40.09	37.26	36.23	32.83
-0.36	21.06	38.20	26.05	21.63	21.40
4.94	23.86	38.93	0.00	0.00	0.00
1.78	23.10	40.09	27.86	23.37	23.26
22.60	34.15	43.42	35.19	32.18	32.96
1.31	30.29	53.26	35.60	29.09	29.57
21.02	40.85	56.61	38.41	31.66	35.33
46.27	42.26	39.01	34.31	32.59	36.44

FIGURE A-1. ACTUAL INTERSECTION INPUT (FREE FORMAT)

TABLE A-2. MISCELLANEOUS INPUT

Gun Location (yd)	
East	North
62.720	34.745

Estimated Impacts (yd)		
Round	Range	Drift
1	16817.82	39.46
2	16621.34	48.34
3	16848.13	34.53
4	16767.67	21.52
5	16886.78	23.86
6	16864.80	23.31
7	16833.95	33.56
8	16876.82	29.93
9	16954.45	36.87
10	16838.30	37.73

Theodolite Locations (yd)		
Number	East	North
1	5361.454	-7453.563
2	6201.392	-8514.340
3	6636.112	-10231.154
4	15623.424	-18920.402

```
62.720, 34.745
16817.82, 39.46
16621.34, 48.34
16848.13, 34.53
16767.67, 21.52
16886.78, 23.86
16864.80, 23.31
16833.95, 33.56
16876.82, 29.93
16954.45, 36.87
16838.30, 37.73
5361.454, 6201.392, 6636.112, 15623.424
```

FIGURE A-2. ACTUAL MISCELLANEOUS INPUT (FREE FORMAT)

ANGLE OF GUN LOF FROM NORTH 128.028
GUN LOC IN FIXED FRAME E 62.720 N 34.745

Estimated Impact (yd)

Round	Range	Drift	East	North
1	16817.820	39.460	13334.591	-10294.731
2	16621.340	48.340	13185.292	-10166.695
3	16848.130	34.530	13355.429	-10317.287
4	16767.670	21.520	13284.035	-10277.968
5	16886.780	23.860	13379.301	-10349.502
6	16864.800	23.310	13361.648	-10336.394
7	16833.950	33.560	13343.662	-10309.315
8	16876.820	29.930	13375.195	-10338.585
9	16954.450	36.870	16440.620	-10380.942
10	16838.300	37.730	13349.657	-10308.710

Theodolite in Fixed Frame (yd)

Number	East	North
1	5361.454	-7453.563
2	6201.392	-8514.340
3	6636.112	-10231.154
4	15623.424	-18920.402

Intersections (yd)

Round	Theodolite Combinations	Range	Drift	East	North
1	(1,2)	16866.540	22.600	13362.581	-10338.026
1	(1,3)	16808.210	41.450	13328.247	-10287.243
1	(2,3)	16788.180	56.540	13321.765	-10263.017
1	(2,4)	16823.820	41.100	13340.327	-10297.135
1	(3,4)	16816.210	35.430	13330.840	-10296.914
1	(1,4)	16819.430	37.830	13334.855	-10297.007
2	(1,2)	16714.240	10.570	13235.202	-10253.678
2	(1,3)	16645.800	33.220	13195.245	-10193.674
2	(2,3)	16622.410	51.420	13188.032	-10164.928
2	(2,4)	16619.650	52.650	13186.616	-10162.259
2	(3,4)	16620.270	53.090	13187.375	-10162.295
2	(1,4)	16609.430	45.250	13174.007	-10161.792
3	(1,2)	16906.960	12.140	13387.977	-10371.166
3	(1,3)	16858.870	27.670	13359.663	-10329.307
3	(2,3)	16842.280	40.090	13354.246	-10309.303
3	(2,4)	16848.820	37.260	13357.654	-10315.561
3	(3,4)	16847.440	36.230	13355.933	-10315.523
3	(1,4)	16842.900	32.830	13350.262	-10315.404
4	(1,2)	16833.480	-0.360	13322.395	-10335.745
4	(1,3)	16768.040	21.060	13284.043	-10278.558
4	(2,3)	16745.570	38.200	13276.902	-10251.214
4	(2,4)	16773.260	26.050	13291.229	-10277.842

FIGURE A-3. THEODOLITE OUTPUT (REPORT FORMAT)

4	(3,4)	16767.300	21.630	13283.811	-10277.653
4	(1,4)	16767.000	21.400	13283.433	-10277.650
5	(1,2)	16945.470	4.940	13413.876	-10400.561
5	(1,3)	16886.780	23.860	13379.301	-10349.502
5	(2,3)	16866.560	38.930	13372.657	-10325.175
5	(2,4)	0.000	0.000	62.720	34.745
5	(3,4)	0.000	0.000	62.720	34.745
5	(1,4)	0.000	0.000	62.720	34.745
6	(1,2)	16930.930	1.780	13400.476	-10394.093
6	(1,3)	16864.970	23.100	13361.653	-10336.665
6	(2,3)	16842.280	40.090	13354.246	-10309.303
6	(2,4)	16870.590	27.860	13369.012	-10336.377
6	(3,4)	16864.620	23.370	13361.543	-10336.236
6	(1,4)	16864.480	23.260	13361.365	-10336.237
7	(1,2)	16866.540	22.600	13362.581	-10338.026
7	(1,3)	16830.800	34.150	13341.544	-10306.910
7	(2,3)	16818.450	43.420	13337.526	-10292.000
7	(2,4)	16837.460	35.190	13347.431	-10310.194
7	(3,4)	16833.430	32.180	13342.402	-10310.082
7	(1,4)	16834.480	32.960	13343.710	-10310.115
8	(1,2)	16964.900	1.310	13426.945	-10415.390
8	(1,3)	16874.900	30.290	13373.904	-10337.118
8	(2,3)	16844.090	53.260	13363.785	-10300.044
8	(2,4)	16885.150	35.600	13385.249	-10339.250
8	(3,4)	16876.500	29.090	13374.425	-10339.049
8	(1,4)	16877.130	29.570	13375.217	-10339.059
9	(1,2)	17001.860	21.020	13468.201	-10422.634
9	(1,3)	16939.540	40.850	13431.327	-10368.621
9	(2,3)	16918.080	56.610	13424.132	-10342.987
9	(2,4)	16960.920	38.410	13446.665	-10383.714
9	(3,4)	16952.040	31.660	13435.512	-10383.561
9	(1,4)	16956.870	35.330	13441.577	-10383.646
10	(1,2)	16817.230	46.270	13338.321	-10289.003
10	(1,3)	16829.680	42.260	13345.658	-10299.832
10	(2,3)	16834.010	39.010	13347.066	-10305.059
10	(2,4)	16844.880	34.310	13352.733	-10315.458
10	(3,4)	16842.580	32.590	13349.862	-10315.396
10	(1,4)	16847.730	36.440	13356.291	-10315.536

FIGURE A-3. THEODOLITE OUTPUT (REPORT FORMAT) (Continued)

```
THEODOLITE Variance over all rounds 0.6487E-05 for THEODOLITE 1
THEODOLITE Variance over all rounds 0.1072E-04 for THEODOLITE 2
THEODOLITE Variance over all rounds 0.2930E-05 for THEODOLITE 3
THEODOLITE Variance over all rounds 0.1870E-06 for THEODOLITE 4

DRFT-CONFIDENCE for RD 1 is 0.1344 for THEODOLITES (1,2)
DRFT-CONFIDENCE for RD 1 is 0.3612 for THEODOLITES (1,3)
DRFT-CONFIDENCE for RD 1 is 0.1445 for THEODOLITES (2,3)
DRFT-CONFIDENCE for RD 1 is 0.7759 for THEODOLITES (2,4)
DRFT-CONFIDENCE for RD 1 is 0.9906 for THEODOLITES (3,4)
DRFT-CONFIDENCE for RD 1 is 0.7898 for THEODOLITES (1,4)

RNGE-CONFIDENCE for RD 1 is 0.0482 for THEODOLITES (1,2)
RNGE-CONFIDENCE for RD 1 is 0.2505 for THEODOLITES (1,3)
RNGE-CONFIDENCE for RD 1 is 0.1600 for THEODOLITES (2,3)
RNGE-CONFIDENCE for RD 1 is 0.6366 for THEODOLITES (2,4)
RNGE-CONFIDENCE for RD 1 is 0.9560 for THEODOLITES (3,4)
RNGE-CONFIDENCE for RD 1 is 0.6617 for THEODOLITES (1,4)
```

FIGURE A-3. THEODOLITE OUTPUT (REPORT FORMAT) (Continued)

```
ENTER # OF ROUNDS
10
NO CUTS (3 OR 6)
6
ENTER ANGLE OF GUN LINE OF FIRE FROM NORTH
128.028
ENTER WHICH ROUND (SAMPLE PT), 1 TO 10, DESIRED
1
ENTER CONFIDENCE DISTANCE FROM SAMPLE MEDIAN
20
ENTER # REPLICATIONS IN GAUSSIAN SIMULATION
10000
```

FIGURE A-4. INTERACTIVE INPUT

APPENDIX B

THEODOLITE PROGRAM CODE

```
      PROGRAM THEODOLITE
C REPLACED THE CONFIDENCE INTERVAL OF ORDER STAT WITH READ IN FIXED
C CONFIDENCE INTERVAL.  ALSO, IMPROVED EQTN FOR SAMPLE VAR.
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BLK1/XCUT(6,10),YCUT(6,10)
      COMMON/BLK4/ XCUTI(6,10),YCUTI(6,10),DMEDG(2,10),DMED(2,10)
      COMMON/BLK2/ZG(6,2),Z(6,2),VAR(4),TH(4),T(4),THMEAN(4,10),PROB(6)
      COMMON/BLK3/THEODX(4),THEODY(4),SD(4)
      DIMENSION VARA(4),VARB(4),PNUM(6),PDEN(6)
      CHARACTER*5 COMBIN(6)
      DATA COMBIN/'(1,2)','(1,3)','(2,3)','(2,4)','(3,4)','(1,4)'/
      DO I=1,6
       PNUM(I)=0.
       PDEN(I)=0.
      ENDDO
      PI=3.141592654
      PI3D2=3.*PI/2.
      RADFAC=PI/180.
      HALFPI=PI/2.
      DO I=1,4
        VARA(I)=0.
        VARB(I)=0.
      ENDDO
C     NRDS=2
      WRITE(6,*)'ENTER # OF ROUNDS'
      READ(5,*) NRDS
    2 WRITE(6,*)'NO CUTS (3 OR 6)'
      READ(5,*) NCUTS
      IF(NCUTS.NE.3.AND.NCUTS.NE.6)THEN
        WRITE(6,*)'NUMBER NCUTS WRONG'
        GO TO 2
      ENDIF
C CLEAR THE X & Y ENTRIES BEFORE USE.
      DO I=1,4
        DO J=1,4
        XCUT(I,J)=0.
        YCUT(I,J)=0.
        ENDDO
        VAR(I)=0.
      ENDDO
C NEED LINE OF SIGHT OF GUN TO DO ROTATION OF AXIS FORM RANGE/DEFL
C TO A FIXED FRAME.
      WRITE(6,*)'ENTER ANGLE OF GUN LINE OF FIRE FROM NORTH'
      READ(5,*) GUNLOFD
      WRITE(8,80)GUNLOFD
   80 FORMAT(1X,'ANGLE OF GUN LOF FROM NORTH',2X,F8.3)
C TAKE 180 DEGREE MINUS GUNLOFD SINCE LINE OF FIRE IS GIVEN AS DEGREES
C CLOCKWISE FROM THE Y AXIS OF FIXED FRAME,WHICH IS NORTH, AND THE ANGLE
C COUNTERCLOCKWISE FROM FIXED FRAME X AXIS (EAST) IS MORE DESIRABLE.
```

FIGURE B-1. THEODOLITE PROGRAM CODE

```
      GUNLOFD=90.0-GUNLOFD
      GUNLOF=GUNLOFD*RADFAC
      GUNNEG=-GUNLOF
C NEED GUN LOCATION W.R.T. FIXED FRAME IN ORDER TO CALCULATE IMPACTS
C AND SAMPLE MEDIAN W.R.T. FIXED FRAME BY TREATING GUN LOC. AS A BIAS.
C     WRITE(6,*) 'ENTER GUN LOCATION IN FIXED FRAME FOR X,Y RESP.'
      READ(2,*)GUNLOCX,GUNLOCY
      WRITE(8,84)GUNLOCX,GUNLOCY
  84  FORMAT(1X,'GUN LOC IN FIXED FRAME E ',F10.3,' N ',F10.3)
      WRITE(8,*)
C READ FILE FOR 3 OR 6 XCUTS FOR THEODOLITES (1,2),(1,3),(2,3),(2,4),
C (3,4),(1,4) RESPECTIVELY. USE 0 FOR LAST 3 IF ONLY 3 CUTS
      DO I=1,NRDS
      READ(1,*) (XCUT(J,I),J=1,6)
C     WRITE(8,*)'XCUTS IE CUTS IN RANGE',(XCUT(J,I),J=1,6)
      ENDDO
C READ FILE FOR 3 OR 6 YCUTS FOR THEODOLITES (1,2),(1,3),(2,3),(2,4),
C (3,4),(1,4) RESPECTIVELY. USE 0 FOR LAST 3 IF ONLY 3 CUTS
      DO I=1,NRDS
      READ(1,*) (YCUT(J,I),J=1,6)
C     WRITE(8,*)'YCUTS IE CUTS IN DRIFT',(YCUT(J,I),J=1,6)
      ENDDO
C ROTATE AND TRANSLATE IMPACTS INTERSECTIONS AND SAMPLE MEDIANS (WHICH
C REPRESENT ESTIMATED IMPACTS) TO THE FIXED FRAME.
      DO I=1,NRDS
        DO J=1,6
        CALL ROTATE(GUNLOF,XCUT(J,I),YCUT(J,I),XCUTI(J,I),YCUTI(J,I))
        XCUTI(J,I)=XCUTI(J,I)+GUNLOCX
        YCUTI(J,I)=YCUTI(J,I)+GUNLOCY
        ENDDO
      ENDDO
C      WRITE(6,*)'ENTER ESTIMATED IMPACT POINTS X,Y RESP.'
       WRITE(8,*)
       WRITE(8,*)'ESTIMATED IMPACT (YDS)'
       WRITE(8,100)
 100   FORMAT(1X'ROUND',8X,'RANGE',8X,'DRIFT',9X,'EAST',8X,'NORTH')
      DO I=1,NRDS
        READ(2,*) DMEDG(1,I),DMEDG(2,I)
        CALL ROTATE(GUNLOF,DMEDG(1,I),DMEDG(2,I),DMED(1,I),DMED(2,I))
        DMED(1,I)=DMED(1,I)+GUNLOCX
        DMED(2,I)=DMED(2,I)+GUNLOCY
C       WRITE(6,*)'EST IMPACT EAST/NORTH',I,DMED(1,I),DMED(2,I)
C       WRITE(8,*)'EST IMPACT EAST/NORTH',I,DMED(1,I),DMED(2,I)
        WRITE(8,110)I,DMEDG(1,I),DMEDG(2,I),DMED(1,I),DMED(2,I)
 110    FORMAT(I5,4X,F10.3,3X,F10.3,3X,F10.3,3X,F10.3)
      ENDDO
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
C READ THEODOLITE LOCATIONS IN FIXED FRAME.
C     WRITE(6,*)'READ 4 THEODOLITE LOCATIONS IN FIXED X FRAME'
C     WRITE(6,*)'IF ONLY 3 EXISTS TYPE ZERO FOR 4TH ONE'
      READ(2,*) (THEODX(I),I=1,4)
      WRITE(8,*)
      WRITE(8,*)'THEODOLITE IN FIXED FRAME (YDS)'
      WRITE(8,200)
 200  FORMAT(1X,'NUMBER',8X,'EAST',8X,'NORTH')
C      WRITE(6,*)'READ 4 THEODOLITE LOCATIONS IN FIXED Y FRAME'
C     WRITE(6,*)'IF ONLY 3 EXISTS TYPE ZERO FOR 4TH ONE'
      READ(2,*) (THEODY(I),I=1,4)
      IF(NCUTS.EQ.3)THEN
          NTHEOD=3
      ELSE
          NTHEOD=4
      ENDIF
      DO IWRITE=1,NTHEOD
        WRITE(8,210)IWRITE,THEODX(IWRITE),THEODY(IWRITE)
 210    FORMAT(I5,4X,F10.3,3X,F10.3)
      ENDDO
C READ WHICH ROUND OUT OF 1,...,10 THAT CONFIDENCE INTERVAL DESIRED.
C READ SUBSCRIPTS OF LOWER & UPPER CONFIDENCE LIMIT RESP. FOR X AND Y.
C KPT GIVES THE ROUND # (1 TO 10) WHERE ANALYSIS IS DESIRED.
      WRITE(6,*) 'ENTER WHICH ROUND (SAMPLE PT), 1 TO 10, DESIRED.'
      READ(5,*)KPT
      WRITE(6,*)'ENTER CONFIDENCE DISTANCE IN YDS FROM SAMPLE MEDIAN'
      READ(5,*)EPSI
C CALCULATE THE MEANS OF THE THEODOLITE FOR EACH OF 10 ROUNDS.
      DO I=1,NRDS
C F1 & F2 USED TO CHECK DIV BY 0.
          F1=DMED(2,I)-THEODY(1)
          F2=DMED(1,I)-THEODX(1)
          IF(F1.GT.0.AND.F2.EQ.0)THMEAN(1,I)=PI/2
          IF(F1.LT.0.AND.F2.EQ.0)THMEAN(1,I)=-PI/2
          IF(F1.NE.0.AND.F2.EQ.0)GO TO 60
          THMEAN(1,I)=ATAN((DMED(2,I)-THEODY(1))/(DMED(1,I)-THEODX(1)))
C SINCE ATAN IS PRINCIPLE ARCTAN,IF ARGUMENT IN 2ND OR 3RD QUADRANT,
C PI MUST BE ADDED TO ENSURE RESULTING ANGLE IN 2ND OR 3RD QUADRANT.
          IF(F1.GT.0.0.AND.F2.LT.0.0)THMEAN(1,I)=THMEAN(1,I)+PI
          IF(F1.LT.0.0.AND.F2.LT.0.0)THMEAN(1,I)=THMEAN(1,I)+PI
  60      F1=DMED(2,I)-THEODY(2)
          F2=DMED(1,I)-THEODX(2)
          IF(F1.GT.0.AND.F2.EQ.0)THMEAN(2,I)=PI/2
          IF(F1.LT.0.AND.F2.EQ.0)THMEAN(2,I)=-PI/2
          IF(F1.NE.0.AND.F2.EQ.0)GO TO 62
        THMEAN(2,I)=ATAN((DMED(2,I)-THEODY(2))/(DMED(1,I)-THEODX(2)))
          IF(F1.GT.0.0.AND.F2.LT.0.0)THMEAN(2,I)=THMEAN(2,I)+PI
          IF(F1.LT.0.0.AND.F2.LT.0.0)THMEAN(2,I)=THMEAN(2,I)+PI
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
 62       F1=DMED(2,I)-THEODY(3)
          F2=DMED(1,I)-THEODX(3)
          IF(F1.GT.0.AND.F2.EQ.0)THMEAN(3,I)=PI/2
          IF(F1.LT.0.AND.F2.EQ.0)THMEAN(3,I)=-PI/2
          IF(F1.NE.0.AND.F2.EQ.0)GO TO 64
        THMEAN(3,I)=ATAN((DMED(2,I)-THEODY(3))/(DMED(1,I)-THEODX(3)))
          IF(F1.GT.0.0.AND.F2.LT.0.0)THMEAN(3,I)=THMEAN(3,I)+PI
          IF(F1.LT.0.0.AND.F2.LT.0.0)THMEAN(3,I)=THMEAN(3,I)+PI
 64       F1=DMED(2,I)-THEODY(4)
          F2=DMED(1,I)-THEODX(4)
          IF(F1.GT.0.AND.F2.EQ.0)THMEAN(4,I)=PI/2
          IF(F1.LT.0.AND.F2.EQ.0)THMEAN(4,I)=-PI/2
          IF(F1.NE.0.AND.F2.EQ.0)GO TO 66
        THMEAN(4,I)=ATAN((DMED(2,I)-THEODY(4))/(DMED(1,I)-THEODX(4)))
          IF(F1.GT.0.0.AND.F2.LT.0.0)THMEAN(4,I)=THMEAN(4,I)+PI
          IF(F1.LT.0.0.AND.F2.LT.0.0)THMEAN(4,I)=THMEAN(4,I)+PI
 66   ENDDO
C CALCULATE THE SAMPLE VARIANCES OF THE THEODOLITES. VAR(I) IS THE ESTI-
C MATED VAR OF ITH THEOD.
      KNT1=0
      KNT2=0
      KNT3=0
      KNT4=0
      WRITE(8,*)
      WRITE(8,*) 'INTERSECTIONS (YDS)'
      WRITE(8,300)
 300  FORMAT(1X,'ROUNDS',2X,'THEODOLITE COMBINATIONS',
     &8X,'RANGE',9X,'DRFT',9X,'EAST',8X,'NORTH')
      DO I=1,NRDS
C       WRITE(6,*)'INTERSECTION-EAST',I,(XCUTI(K,I),K=1,6)
C       WRITE(6,*)'INTERSECTION-NORTH',I,(YCUTI(K,I),K=1,6)
C       WRITE(6,310)I,XCUT(1,I),YCUT(1,I)
C       WRITE(6,315)I,XCUT(2,I),YCUT(2,I)
C       WRITE(6,320)I,XCUT(3,I),YCUT(3,I)
C       IF(NCUTS.GT.3)THEN
C         WRITE(6,325)I,XCUT(4,I),YCUT(4,I)
C         WRITE(6,330)I,XCUT(5,I),YCUT(5,I)
C         WRITE(6,335)I,XCUT(6,I),YCUT(6,I)
C       ENDIF
        WRITE(8,310)I,XCUT(1,I),YCUT(1,I),XCUTI(1,I),YCUTI(1,I)
        WRITE(8,315)I,XCUT(2,I),YCUT(2,I),XCUTI(2,I),YCUTI(2,I)
        WRITE(8,320)I,XCUT(3,I),YCUT(3,I),XCUTI(3,I),YCUTI(3,I)
        IF(NCUTS.GT.3) THEN
          WRITE(8,325)I,XCUT(4,I),YCUT(4,I),XCUTI(4,I),YCUTI(4,I)
          WRITE(8,330)I,XCUT(5,I),YCUT(5,I),XCUTI(5,I),YCUTI(5,I)
          WRITE(8,335)I,XCUT(6,I),YCUT(6,I),XCUTI(6,I),YCUTI(6,I)
        ENDIF
        WRITE(8,*)
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
310    FORMAT(1X,I5,21X,'(1,2)',3X,F10.3,3X,F10.3,3X,F10.3,3X,F10.3)
315    FORMAT(1X,I5,21X,'(1,3)',3X,F10.3,3X,F10.3,3X,F10.3,3X,F10.3)
320    FORMAT(1X,I5,21X,'(2,3)',3X,F10.3,3X,F10.3,3X,F10.3,3X,F10.3)
325    FORMAT(1X,I5,21X,'(2,4)',3X,F10.3,3X,F10.3,3X,F10.3,3X,F10.3)
330    FORMAT(1X,I5,21X,'(3,4)',3X,F10.3,3X,F10.3,3X,F10.3,3X,F10.3)
335    FORMAT(1X,I5,21X,'(1,4)',3X,F10.3,3X,F10.3,3X,F10.3,3X,F10.3)
C BOTH XCUT & YCUT MUST BE 0 FOR IT TO 'NOT COUNT' AS AN ENTRY.
       IF(XCUT(1,I).NE.0.0.OR.YCUT(1,I).NE.0.0)THEN
C F1 & F2 USED TO CHECK DIV BY 0.
         F1=YCUTI(1,I)-THEODY(1)
         F2=XCUTI(1,I)-THEODX(1)
         IF(F1.GT.0.AND.F2.EQ.0)VAR0=PI/2
         IF(F1.LT.0.AND.F2.EQ.0)VAR0=-PI/2
         IF(F1.NE.0.AND.F2.EQ.0)GO TO 50
C VAR0 REPRESENTS ANGLE WHOSE RAYS INTERSECT AT (XCUTI,YCUTI). THIS
C BE USED TO COMPUTE APPROXIMATE SAMPLE VARIANCE.
         VAR0=ATAN((YCUTI(1,I)-THEODY(1))/(XCUTI(1,I)-THEODX(1)))
C SINCE ATAN IS PRINCIPLE ARCTAN,IF ARGUMENT IN 2ND OR 3RD QUADRANT,
C PI MUST BE ADDED TO ENSURE RESULTING ANGLE IN 2ND OR 3RD QUADRANT.
         IF(F1.GT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
         IF(F1.LT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
 50      KNT1=KNT1+1
C VARA AND VARB ARE USED TO ESTIMATE APPROX. SAMPLE VARIANCE. SINCE
C THE OBSERVATION MEANS DIFFER FOR EACH ROUND, ONLY AN APPROX. SAMPLE
C VARIANCE CAN BE DERIVED. THIS APPROX. SAMPLE VARIANCE IS THE ACTUAL
C SAMPLE VARIANCE OF THE RANDOM VARIABLE, VAR0-THMEAN, WHERE VAR0 IS
C DEFINED IN A COMMENT STATEMENT ABOVE AND THMEAN IS THE SAMPLE MEDIAN
C OF A GIVEN ROUND AS ESTIMATED BY THE PROGRAM THAT THIS PROGRAM IS
C CHECKING. NOTE THAT E(VAR0-THMEAN)=E(VAR0)-E(THMEAN)=0, AND SO ONE
C CAN LEGALLY CONSIDER THE SAMPLE VARIANCE OF THIS RANDOM VARIABLE.
         VARA(1)=VARA(1)+(VAR0-THMEAN(1,I))**2
         VARB(1)=VARB(1)+VAR0-THMEAN(1,I)
         F1=YCUTI(1,I)-THEODY(2)
         F2=XCUTI(1,I)-THEODX(2)
         IF(F1.GT.0.AND.F2.EQ.0)VAR0=PI/2
         IF(F1.LT.0.AND.F2.EQ.0)VAR0=-PI/2
         IF(F1.NE.0.AND.F2.EQ.0)GO TO 52
         VAR0=ATAN((YCUTI(1,I)-THEODY(2))/(XCUTI(1,I)-THEODX(2)))
         IF(F1.GT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
         IF(F1.LT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
 52      KNT2=KNT2+1
         VARA(2)=VARA(2)+(VAR0-THMEAN(2,I))**2
         VARB(2)=VARB(2)+VAR0-THMEAN(2,I)
       ENDIF
       IF(XCUT(2,I).NE.0.0.OR.YCUT(2,I).NE.0.0)THEN
         F1=YCUTI(2,I)-THEODY(3)
         F2=XCUTI(2,I)-THEODX(3)
         IF(F1.GT.0.AND.F2.EQ.0)VAR0=PI/2
         IF(F1.LT.0.AND.F2.EQ.0)VAR0=-PI/2
         IF(F1.NE.0.AND.F2.EQ.0)GO TO 54
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
         VAR0=ATAN((YCUTI(2,I)-THEODY(3))/(XCUTI(2,I)-THEODX(3)))
         IF(F1.GT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
         IF(F1.LT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
   54    KNT3=KNT3+1
         VARA(3)=VARA(3)+(VAR0-THMEAN(3,I))**2
         VARB(3)=VARB(3)+VAR0-THMEAN(3,I)
       ENDIF
       IF(XCUT(6,I).NE.0.0.OR.YCUT(6,I).NE.0.0)THEN
         F1=YCUTI(6,I)-THEODY(4)
         F2=XCUTI(6,I)-THEODX(4)
         IF(F1.GT.0.AND.F2.EQ.0)VAR0=PI/2
         IF(F1.LT.0.AND.F2.EQ.0)VAR0=-PI/2
         IF(F1.NE.0.AND.F2.EQ.0)GO TO 56
         VAR0=ATAN((YCUTI(6,I)-THEODY(4))/(XCUTI(6,I)-THEODX(4)))
         IF(F1.GT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
         IF(F1.LT.0.0.AND.F2.LT.0.0)VAR0=VAR0+PI
   56   KNT4=KNT4+1
         VARA(4)=VARA(4)+(VAR0-THMEAN(4,I))**2
         VARB(4)=VARB(4)+VAR0-THMEAN(4,I)
         ENDIF
       ENDDO
       VAR(1)=(KNT1*VARA(1))-VARB(1)**2
       VAR(2)=(KNT2*VARA(2))-VARB(2)**2
       VAR(3)=(KNT3*VARA(3))-VARB(3)**2
       IF(NCUTS.EQ.6)VAR(4)=(KNT4*VARA(4))-VARB(4)**2
       VAR(1)=VAR(1)/(KNT1*(KNT1-1))
       VAR(2)=VAR(2)/(KNT2*(KNT2-1))
       VAR(3)=VAR(3)/(KNT3*(KNT3-1))
       IF(NCUTS.EQ.6)VAR(4)=VAR(4)/(KNT4*(KNT4-1))
C IN THE SIMULATION TO GET PROB OF EACH INTERSECTION LESS THAN MEDIAN,
C DEFINE THE FOLLOWING, WHERE K REPRESETS KTH SAMPLE:
C       Z(1,1) REPRESENT XCUTI(1,K)
C       Z(2,1) REPRESENT XCUTI(2,K)
C       Z(3,1) REPRESENT XCUTI(3,K)
C       Z(4,1) REPRESENT XCUTI(4,K)
C       Z(5,1) REPRESENT XCUTI(5,K)
C       Z(6,1) REPRESENT XCUTI(6,K)
C       Z(1,2) REPRESENT YCUTI(1,K)
C       Z(2,2) REPRESENT YCUTI(2,K)
C       Z(3,2) REPRESENT YCUTI(3,K)
C       Z(4,2) REPRESENT YCUTI(4,K)
C       Z(5,2) REPRESENT YCUTI(5,K)
C       Z(6,2) REPRESENT YCUTI(6,K)
C FOLLOWING ESTI. CONFIDENCE INTERVAL PROB  BY MONTE
C CARLO SIMULATION FOR A GIVEN SAMPLE (1 TO 10) AND GIVEN LOWER &
C UPPER X AND Y CONFIDENCE INTERVALS. THE CONFIDENCE INTERVAL IS BETWEEN
C SAMPLE MEDIAN - EPSI TO SAMPLE MED + EPSI. KPT GIVES THE ROUND
C BETWEEN 1 TO 10 THAT CONFIDENCE INTERVAL IS DESIRED.
C
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
C IXY=1 IMPLIES WORKING WITH X AXIS & IXY=2 IMPLIES WORKING WITH Y AXIS.
C IX IS THE SEED TO THE GAUSSIAN RANDOM # GENERATOR TO BE USED (NRNG).
      IX=246174
      WRITE(6,*)'ENTER # REPLICATIONS IN GAUSSAIN SIMULATION'
      READ(5,*)NREPL
      DO IXY=2,1,-1
      IF(IXY.EQ.1)THEN
        LOWXY=LOWX
        HIGHXY=HIGHX
      ELSE
        LOWXY=LOWY
        HIGHXY=HIGHY
      ENDIF
      IF(IXY.EQ.2)THEN
        WRITE(6,*)
        DO ITHEOD=1,NTHEOD
          WRITE(6,400)VAR(ITHEOD),ITHEOD
          WRITE(8,400)VAR(ITHEOD),ITHEOD
        ENDDO
      ENDIF
 400  FORMAT(1X,'THEODOLITE VARIANCE OVER ALL ROUNDS',E11.4,3X,
     &'FOR THEODOLITE  ',I2)
C NRNG GENERATES A STANDARDIZED GAUSSIAN RANDOM NUMBER. IX IS THE SEED,
C TH IS A 4 DIMENSIONED ARRAY OF NUMBERS TO BE GENERATED, REPRESENTING
C THEODOLITE ANGLES FOR 4 THEODOLITES. THE 4 IN THE PARAMETERS OF NRNG
C IMPLIES THAT 4 NUMBERS ARE TO BE GENERATED. THE IERR IS THE ERROR
C MESSAGE. IF NO ERRORS DURING RUN, IERR=0. SEE NSWC LIBR. OF MATH
C SUBROUTINES FOR DETAILED EXPLANATION OF NRNG.
        DO 40 N=1,NREPL
C 22          CALL NRNG(IX,T,4,IERR)
  22          CALL GAUSS(IX,T(1),T(2))
              CALL GAUSS(IX,T(3),T(4))
                DO 45 NN=1,4
                SD(NN)=SQRT(VAR(NN))
                TH(NN)=SD(NN)*T(NN)+THMEAN(NN,KPT)

C TH(NN) NOT ALLOW TO BE 90,270,-90,NOR -270 DEGREES SINCE THESE VALUES
C CAUSES THE TAN TO BE INFINITE. THIS DOES NOT SIGNIFICANTLY EFFECT THE
C DISTRIBUTION OF THE NORMAL RANDOM VAR. SINCE IN THEORY THE R.V. HAS
C 0 PROBABILITY OF OBTAINING ANY SINGLE VALUE.
                IF(TH(NN).EQ.PI3D2.OR.TH(NN).EQ.HALFPI)GO TO 22
                IF(TH(NN).EQ.-PI3D2.OR.TH(NN).EQ.-HALFPI)GO TO 22
  45            CONTINUE
C XFUN,YFUN ARE FUNCTIONS TO COMPUTE THE X & Y CUTS RESP.
                Z(1,2)=YFUN(TH(1),TH(2),1,2)
                Z(2,2)=YFUN(TH(1),TH(3),1,3)
                Z(3,2)=YFUN(TH(2),TH(3),2,3)
C IF NO. OF CUTS EQUAL 3 THE FOLLOWING 3 STATEMENTS ARE NOT NEEDED.
                IF(NCUTS.EQ.6) Z(4,2)=YFUN(TH(2),TH(4),2,4)
                IF(NCUTS.EQ.6) Z(5,2)=YFUN(TH(3),TH(4),3,4)
                IF(NCUTS.EQ.6) Z(6,2)=YFUN(TH(1),TH(4),1,4)
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
                  Z(1,1)=XFUN(Z(1,2),TH(2),2)
                  Z(2,1)=XFUN(Z(2,2),TH(3),3)
                  Z(3,1)=XFUN(Z(3,2),TH(3),3)
C IF NO. OF CUTS EQUAL 3 THE FOLLOWING 3 STATEMENTS ARE NOT NEEDED.
                  IF(NCUTS.EQ.6) Z(4,1)=XFUN(Z(4,2),TH(4),4)
                  IF(NCUTS.EQ.6) Z(5,1)=XFUN(Z(5,2),TH(4),4)
                  IF(NCUTS.EQ.6) Z(6,1)=XFUN(Z(6,2),TH(4),4)
C TRANSFORM BACK TO RANGE/DRIFT FRAME.
                  DO IT=1,3
                   Z(IT,1)=Z(IT,1)-GUNLOCX
                   Z(IT,2)=Z(IT,2)-GUNLOCY
                   CALL ROTATE(GUNNEG,Z(IT,1),Z(IT,2),ZG(IT,1),ZG(IT,2))
                  ENDDO
                  IF(NCUTS.EQ.6)THEN
                   DO IT=3,6
                   Z(IT,1)=Z(IT,1)-GUNLOCX
                   Z(IT,2)=Z(IT,2)-GUNLOCY
                   CALL ROTATE(GUNNEG,Z(IT,1),Z(IT,2),ZG(IT,1),ZG(IT,2))
                   ENDDO
                  ENDIF
C
         KCTR=0
         DO JJ=1,6
          IF(JJ.GE.4.AND.NCUTS.EQ.3)GO TO 30
          ZGG=ZG(JJ,IXY)
          DMEDGG=DMEDG(IXY,KPT)
          IF((ZGG.GE.(DMEDGG-EPSI)).AND.(ZGG.LE.(DMEDGG+EPSI)))THEN
            PNUM(JJ)=PNUM(JJ)+1
            PDEN(JJ)=PDEN(JJ)+1
          ELSE
            PDEN(JJ)=PDEN(JJ)+1
          ENDIF
         ENDDO
   30    CONTINUE
   40    CONTINUE
       DO LH=1,NCUTS
         PROB(LH)=PNUM(LH)/PDEN(LH)
C        WRITE(6,*)'PNUM,PDEN,PROB',PNUM(LH),PDEN(LH),PROB(LH)
C        WRITE(8,*)'PNUM,PDEN,PROB',PNUM(LH),PDEN(LH),PROB(LH)
         PDEN(LH)=0.
         PNUM(LH)=0.
       ENDDO
       WRITE(6,*)
       WRITE(8,*)
       DO LH=1,NCUTS
        IF(IXY.EQ.1)WRITE(6,410)KPT,PROB(LH),COMBIN(LH)
        IF(IXY.EQ.2)WRITE(6,420)KPT,PROB(LH),COMBIN(LH)
        IF(IXY.EQ.1)WRITE(8,410)KPT,PROB(LH),COMBIN(LH)
        IF(IXY.EQ.2)WRITE(8,420)KPT,PROB(LH),COMBIN(LH)
  410  FORMAT(1X,'RNGE-CONFIDENCE FOR RD',I3,' IS ',F10.4,'     FOR THEO
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
     &DOLITES    ',A5)
420   FORMAT(1X,'DRFT-CONFIDENCE FOR RD',I3,' IS ',F10.4,'    FOR THEO
     &DOLITES    ',A5)
      ENDDO
      ENDDO
      END
      SUBROUTINE ROTATE(GUNLOF,XC,YC,XCI,YCI)
      IMPLICIT REAL*8 (A-H,O-Z)
      XCI=XC*COS(GUNLOF)-YC*SIN(GUNLOF)
      YCI=XC*SIN(GUNLOF)+YC*COS(GUNLOF)
      RETURN
      END
      FUNCTION YFUN(THA,THB,NA,NB)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BLK1/XCUT(6,10),YCUT(6,10)
      COMMON/BLK4/ XCUTI(6,10),YCUTI(6,10),DMEDG(2,10),DMED(2,10)
      COMMON/BLK2/ZG(6,2),Z(6,2),VAR(4),TH(4),T(4),THMEAN(4,10),PROB(6)
      COMMON/BLK3/THEODX(4),THEODY(4),SD(4)
      YFUN11=(THEODY(NA)-THEODX(NA)*TAN(THA))*TAN(THB)
      YFUN12=(THEODY(NB)-THEODX(NB)*TAN(THB))*TAN(THA)
      YFUN1=YFUN11-YFUN12
      YFUN2=TAN(THB)-TAN(THA)
      YFUN=YFUN1/YFUN2
      RETURN
      END
      FUNCTION XFUN(Y,THB,NB)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BLK1/XCUT(6,10),YCUT(6,10)
      COMMON/BLK4/ XCUTI(6,10),YCUTI(6,10),DMEDG(2,10),DMED(2,10)
      COMMON/BLK2/ZG(6,2),Z(6,2),VAR(4),TH(4),T(4),THMEAN(4,10),PROB(6)
      COMMON/BLK3/THEODX(4),THEODY(4),SD(4)
      XFUN1=Y-(THEODY(NB)-THEODX(NB)*TAN(THB))
      XFUN=XFUN1/TAN(THB)
      RETURN
      END
      SUBROUTINE GAUSS(ISEED,RNVAR1,RNVAR2)
C
C     SUBROUTINE GAUSS GENERATES TWO INDEPENDENT,
C     UNCORRELATED GAUSSIAN RANDOM VARIABLES.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DATA TWOPI/6.283185/
C
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

```
    1  X1=RAN(ISEED)
       IF(X1.EQ.0.)GO TO 1
       X2=RAN(ISEED)
       X3=SQRT(-2.0*LOG(X1))
       X4=TWOPI*X2
       RNVAR1=X3*COS(X4)
       RNVAR2=X3*SIN(X4)
C
       RETURN
       END
```

FIGURE B-1. THEODOLITE PROGRAM CODE (Continued)

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